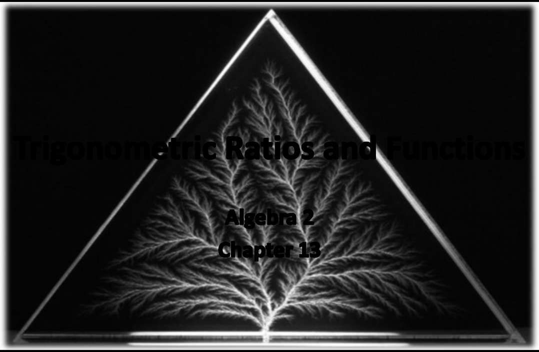

Trigonometric Ratios and Functions

Algebra 2
Chapter 13

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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13.1 Use Trigonometry with Right Triangles

- If you have a right triangle, there are six ratios of sides that are always constant
  - $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
  - $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
  - $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
  - $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$
  - $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
  - $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

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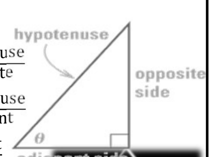
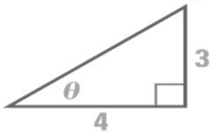
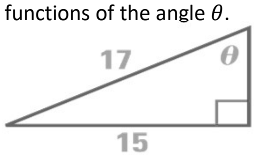
CAH
TOA



13.1 Use Trigonometry with Right Triangles

- Evaluate the six trigonometric functions of the angle θ.



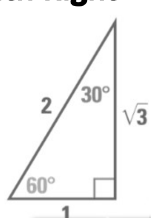


13.1 Use Trigonometry with Right Triangles

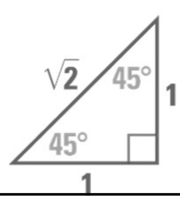
- In a right triangle, θ is an acute angle and $\cos\theta = \frac{7}{10}$. What is $\sin\theta$?

13.1 Use Trigonometry with Right Triangles

- Special Right Triangles
- 30° - 60° - 90°

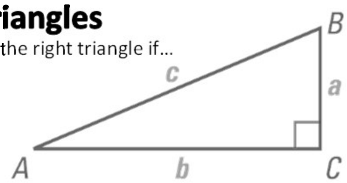


- 45° - 45° - 90°



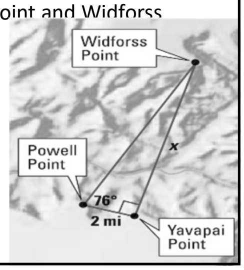
13.1 Use Trigonometry with Right Triangles

- Use the diagram to solve the right triangle if…
- B = 45°, c = 5
- B = 60°, a = 7
- A = 32°, b = 10



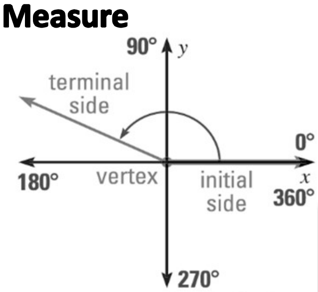
13.1 Use Trigonometry with Right Triangles

- Find the distance between Powell Point and Widforss Point.



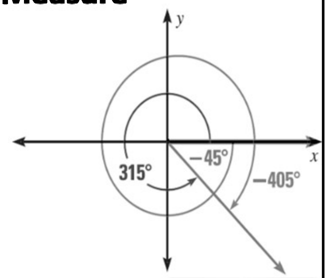
13.2 Define General Angles and Use Radian Measure

- Angles in Standard Position
  - Vertex on origin
  - Initial Side on positive x-axis
  - Measured counterclockwise



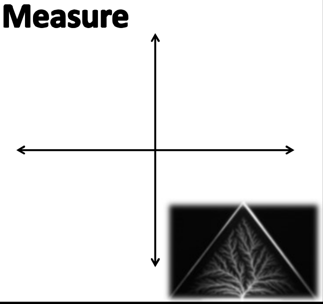
13.2 Define General Angles and Use Radian Measure

- Coterminal Angles
  - Different angles (measures) that have the same terminal side
  - Found by adding or subtracting multiples of 360°



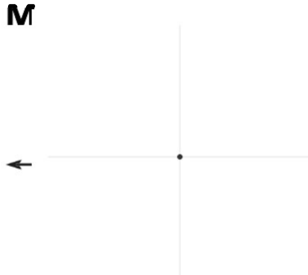
13.2 Define General Angles and Use Radian Measure

- Draw an angle with the given measure in standard position. Then find one positive coterminal angle and one negative coterminal angle.
- 65°
- 300°



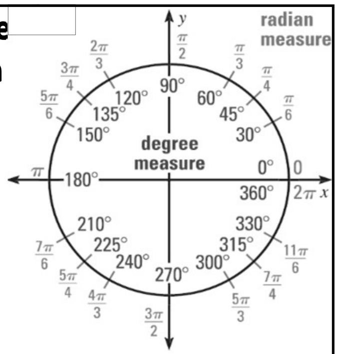
13.2 Define General Angles and Use Radian M

- Radian measure
  - Another unit to measure angles
  - 1 radian is the angle when the arc length = the radius
  - There are 2π radians in a circle



13.2 Define Gene Radian

- To convert between degrees and radians use fact that
  - 180° = π
- Special angles

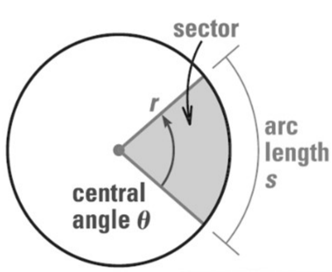


13.2 Define General Angles and Use Radian Measure

- Convert the degree measure to radians, or the radian measure to degrees.
- 135°
- -50°
- $\frac{5\pi}{4}$
- $\frac{\pi}{10}$

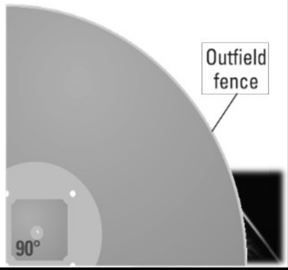
13.2 Define General Angles and Use Radian Measure

- Sector
  - Slice of a circle
- Arc Length
  - $s = r\theta$
  - θ must be in radians!
- Area of Sector
  - $A = \frac{1}{2}r^2\theta$
  - θ must be in radians!



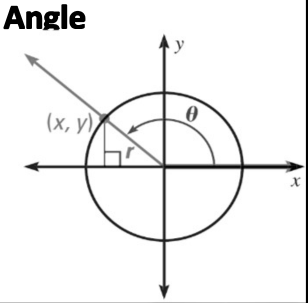
13.2 Define General Angles and Use Radian Measure

- Find the length of the outfield fence if it is 220 ft from home plate.
- Find the area of the baseball field.



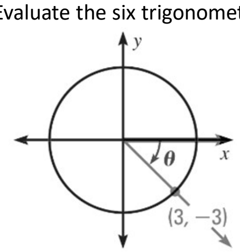
13.3 Evaluate Trigonometric Functions of Any Angle

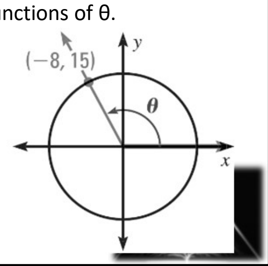
- Think of a point on the terminal side of an angle
- You can draw a right triangle with the x-axis
- $\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$
- $\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$
- $\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$
- Unit Circle
  - r = 1



13.3 Evaluate Trigonometric Functions of Any Angle

- Evaluate the six trigonometric functions of θ.



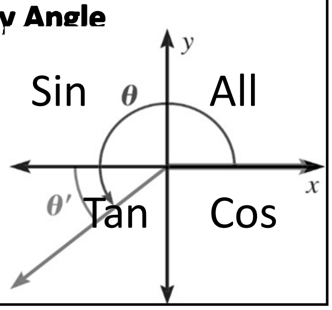


13.3 Evaluate Trigonometric Functions of Any Angle

- Evaluate the six trigonometric functions of θ.
- θ = 180°

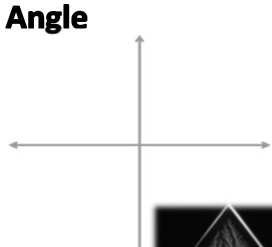
13.3 Evaluate Trigonometric Functions of Any Angle

- Reference Angle
  - Angle between terminal side and x-axis
  - Has the same values for trig functions as 1st quadrant angles
  - You just have to add the negative signs



13.3 Evaluate Trigonometric Functions of Any Angle

- Sketch the angle. Then find its reference angle.
- 150°
- $-\frac{7\pi}{9}$
- Evaluate cos(-60°) without a calculator



13.3 Evaluate Trigonometric Functions of Any Angle

- Estimate the horizontal distance traveled by a Red Kangaroo who jumps at an angle of 8° and with an initial speed of 53 feet per second (35 mph).

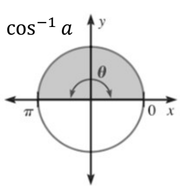
13.4 Evaluate Inverse Trigonometric Functions

- Find an angle whose tangent = 1
- There are many
  - $\frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}$, etc.
- In order to find angles given sides (or x and y) we have to define the functions carefully

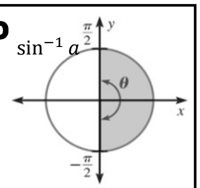
13.4 Evaluate Inverse Trigo Functions

- Inverse Trig Functions
- $\sin^{-1} a = \theta$
  - $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
- $\cos^{-1} a = \theta$
  - $0 \le \theta \le \pi$
- $\tan^{-1} a = \theta$
  - $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

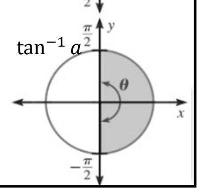
$\cos^{-1} a$



$\sin^{-1} a$



$\tan^{-1} a$



13.4 Evaluate Inverse Trigonometric Functions

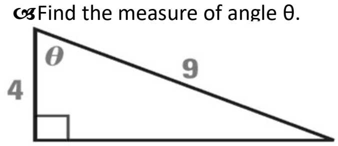
- Evaluate the expression in both radians and degrees.
- $\sin^{-1}\frac{\sqrt{2}}{2}$
- $\cos^{-1}\frac{1}{2}$
- $\tan^{-1} -1$

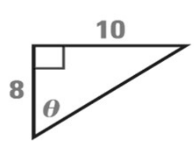
13.4 Evaluate Inverse Trigonometric Functions

- Solve the equation for θ
- $\cos\theta = 0.4; 270° < \theta < 360°$
- $\tan\theta = 4.7; 180° < \theta < 270°$
- $\sin\theta = 0.62; 90° < \theta < 180°$

13.4 Evaluate Inverse Trigonometric Functions

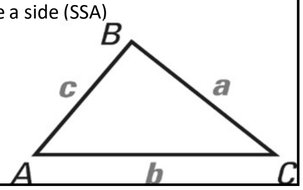
- Find the measure of angle θ.





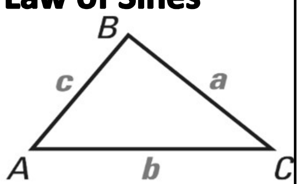
13.5 Apply the Law of Sines

- In lesson 13.1 we solved right triangles
- In this lesson we will solve any triangle if we know
  - 2 Angles and 1 Side (AAS or ASA)
  - 2 Sides and 1 Angle opposite a side (SSA)
- Law of Sines
  - $\frac{\sin}{a} = \frac{\sin}{b} = \frac{\sin}{c}$



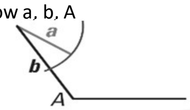
13.5 Apply the Law of Sines

- Solve ΔABC if…
  - A = 51°, B = 44°, c = 11

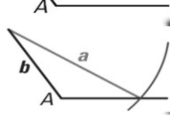


13.5 Apply the Law of Sines

- Indeterminant Case (SSA)
  - Maybe no triangle, one triangle, or two triangles
- In these examples, you know a, b, A
- If A > 90° and…
  - a ≤ b → no triangle

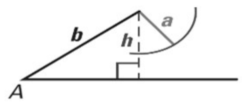


  - a > b → 1 triangle

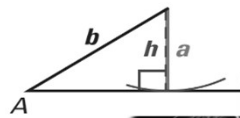


13.5 Apply the Law of Sines

- A < 90° and… (h = b sin A)
  - h > a → no triangle

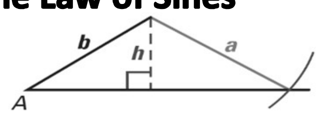


  - h = a → one triangle

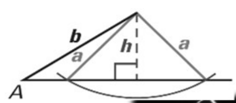


13.5 Apply the Law of Sines

- a ≥ b → one triangle



- h < a < b → two triangles

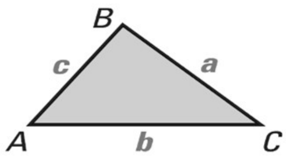


13.5 Apply the Law of Sines

- Solve ΔABC
  - A = 122°, a = 18, b = 12
  - A = 36°, a = 9, b = 12

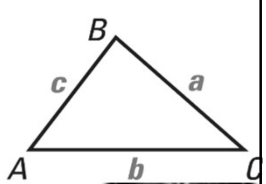
13.5 Apply the Law of Sines

- Area of Triangle
  - $Area = \frac{1}{2}bh$
  - $h = c \sin A$
  - $Area = \frac{1}{2}bc \sin A$
- Find the area of ΔABC with…
  - a = 10, b = 14, C = 46°



13.6 Apply the Law of Cosines

- When you need to solve a triangle and can't use Law of Sines, use Law of Cosines
  - 2 Sides and Included angle (SAS)
  - 3 Sides (SSS)
- Law of Cosines
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $b^2 = a^2 + c^2 - 2ac \cos B$
  - $c^2 = a^2 + b^2 - 2ab \cos C$



13.6 Apply the Law of Cosines

- Solve ΔABC if…
  - a = 8, c = 10, B = 48°
  - a = 14, b = 16, c = 9

13.6 Apply the Law of Cosines

- Heron's Area Formula
  - $Area = \sqrt{s(s-a)(s-b)(s-c)}$
    - Where $s = \frac{1}{2}(a+b+c)$
- Find the area of ΔABC